Interest Rate Derivatives

An Introduction to Interest Rate Swaps, LIBOR rates, Caps and Floors

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Interest Rate Swaps

Libor Rates - a Mathematical approach

First of all, what is Libor? Libor, or technically LIBOR, is an acronym for London InterBank Offered Rate. For the sake of simplicity, I will denote the LIBOR rate from current time t to $t+h$ as $L_t(t, t+h)$. LIBOR is the rate at which banks can deposit or borrow money at; for example Bank A can borrow $\$K$ at time t from Bank B and at maturity (when h units of time have passed) pay back $\$K(1 + hL_t(t, t+h))$. Our variable h is measured in years and thus a LIBOR rate over 3 months has $h = 0.25$ and a LIBOR rate over 6 months has $h = 0.5$. These two lengths are the most common types of LIBOR.

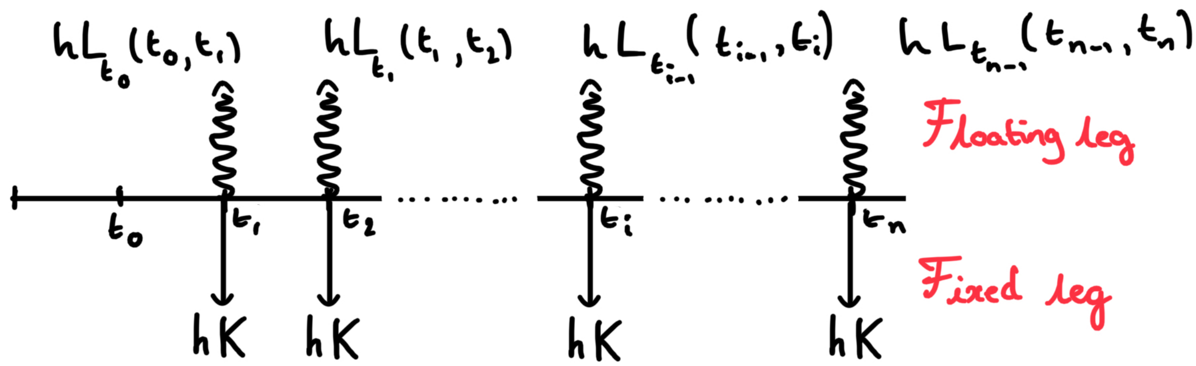
How is LIBOR calculated?

Each LIBOR is calculated by gathering the rate at which a number of banks borrow and lend out money, discarding the top and bottom 25% of values and then taking the mean of the remaining 50%.

A Brief Introduction to Swaps

Interest Rate swaps are the most widely traded of all over the counter (OTC) derivatives contracts. Swaps allow for institutions to trade changes in interest rates, or to manage exposure to swings in interest rates.

A swap is an agreement between two parties to exchange a series of cashflows on pre-determined dates. A swap, obviously, has a start date, t_O and an end date, known as the maturity, t_N. We can denote the series of dates by $t_1, t_2, ..., t_N$.



In a vanilla (standard) swap, we have that $t_{i+1} = t_i + h$. The fixed leg is made up of n payments of hK, where K is the fixed rate accrued over the period from t_i to $t_i + \alpha$ and the floating leg consist of payments of $hL_{t_i}(t_i, t_{i+1})$ or from our previous assumption, $hL_{t_i}(t_i, t_i + h)$. In short, the holder of the fixed leg side of the swap pays a rate that was fixed at the start of the swap, and the holder of the floating leg pays a 'floating' rate equal to LIBOR.

Other Interest Rate Derivatives

What are Interest Rate Options?

Interest rate Options are the most commonly traded type of Options contract globally, playing a huge role in the financial markets. Over the next couple of pages, I will introduce the different types of interest rate Options (Caps, Floors and Swapations)

What are Caps?

Caps are a collection of caplets, which are essentially a guarantee that interest rates will not rise to a certain amount. The payoff from a Caplet struck at some rate K_c is given by:

$$P = max(\text{current LIBOR rate} - K_c, 0) * \text{principal} * \frac{\#\text{number of days until maturity}}{360}$$

For example, say we have a loan with a variable interest rate and let the current LIBOR rate be 6%. We believe that it will rise to a value above this so we want to hedge this risk and thus we buy a 90-day caplet struck at 6%. Say we are borrowing $1,000,000 and the LIBOR rate moves up to 7%. The payoff from having this 90-day caplet is:

$$P = max(7\% - 6\%, 0) * 1,000,000 * \frac{90}{360} = \$2,500$$

Caplets are shorter term investments and if we wanted to hedge the risk of an upward movement in LIBOR rates for a longer time, we can buy a series of caplets. Combining these together, we get a cap.

The Black-76 Formula

The Black-76 formula is a method of pricing a Caplet:

$$C(t) = \frac{N\tau}{1 + F\tau} e^{-r(T-t)}(FN(d_1) - KN(d_2))$$

Where:

- N is the principal,
- τ is the tenor, the time left until the contract expires,
- F is the implied forward rate between time t and the caplet's maturity,
- $d_1 = \frac{ln(\frac{F}{K}) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$,
- $d_2 = d_1 - \sigma\sqrt{T}$
- r is the risk-free interest rate.

Let's look at pricing a caplet through an example: Suppose we have a Caplet, with six months to expiry on a 182-day forward rate and a face value of 100 million. The six-month forward rate is 8% (with act/360 as day-count), the strike is 8%, the risk-free interest rate 7%, and the volatility of the forward rate 28% per annum. We then have:

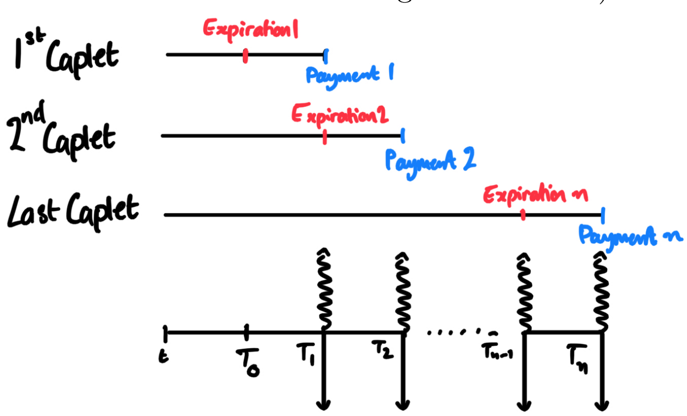
- $F = 0.08$,
- $K = 0.08$,
- $T = 0.5$,
- $r = 0.07$,
- $\sigma = 0.28$,
- $d_1 = \frac{ln(\frac{0.08}{0.08})+0.5*\frac{0.28^2}{2}}{0.28\sqrt{0.5}} = 0.0990$
- $d_2 = 0.0990 - 0.28\sqrt{0.5} = -0.0990$
- $N(x)$ is just the cumulative distribution function of a normal distribution with mean 0 and variance 1, thus $N(d_1) = 0.5394$ and $N(d_2) = 0.4606$.

Plugging these in:

$$C(t) = \frac{10^9 * \frac{182}{360}}{1 + 0.08 * \frac{182}{360}} e^{-0.07*0.05}(0.08 * 0.5394 - 0.08 * 0.4606) = 295.995$$

Caplets and Floorlets

Caplets are, as mentioned before, tools for mitigating, or hedging, the risk of rising interest rates. Another way to put this is a caplet is a call on LIBOR rates. A cap is a portfolio consisting of a series of caplets. A floorlet is the compliment of the caplet, it is a put on LIBOR rates. If one purchases a floorlet, one is essentially betting that LIBOR rates will fall. Similar to a cap, a floor is a portfolio of a series of floorlets. A useful strategy with caplets and floorlets is known as a cap-floor straddle. A cap-floor straddle is the name given to a portfolio containing of a cap and a floor, both with the exact same strike and equivalent dates. A useful diagram to understand caps (and we can draw a similar diagram for floors) is below.



One would buy a cap-floor straddle when they believe the LIBOR rates will rise or fall by a considerable amount, i.e they are betting that the volatility of the LIBOR rates will increase. One would go short a cap-floor straddle when they believe that the LIBOR rates will not move considerably.